SCED 450.01

TRIGONOMETRIC EQUATIONS

CURRICULUM & TEXTBOOK PROJECT

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SCED 450 CURRICULUM &TEXTBOOK PROJECT

Objective 11.4.4.1: The student will be able to find the solution sets of trigonometric equations.

The extension of our objective is based on the 5th grade math unit, triangles, and also 6th grade angles unit and 7th grade circle and disc unit. More importantly, 8th grade right triangle and Pythogorean Theorem unit will be base for our objective because students will use Pythogorean Theorem while solving trigonometric equations. Also, because in order to use sin x, cos x , tan x and cotan x; students first need to know the ratio of sides of an right triangle. After they comprehend those concept, they can go further while solving trigonometric equations. As a result, they need to learn what is a triangle and what is the relationships between sides and angles of a triangle, and also relationships of circles (unit circle) and triangles. We mean by circles that students first understand the unit circle and find right triangles on the unit circle and use sin x ,cos x etc. Finally, students should know how to solve equations for the trigonometric equations topic.

Intended grade level:

11 th grade

Textbook:

In the new curriculum trigonometric equations unit is included in 11th grade but there is no updated 11th grade mathematics book in terms of the new curriculum. In other words, trigonometric equations unit is not included in 11th grade mathematics book yet. Therefore, we found trigonometric equations unit in 10th grade math book which is prepared in terms of the old curriculum. Trigonometric equations unit is between 212 th and 218 th pages.

Specific Math Idea/Key Understanding:

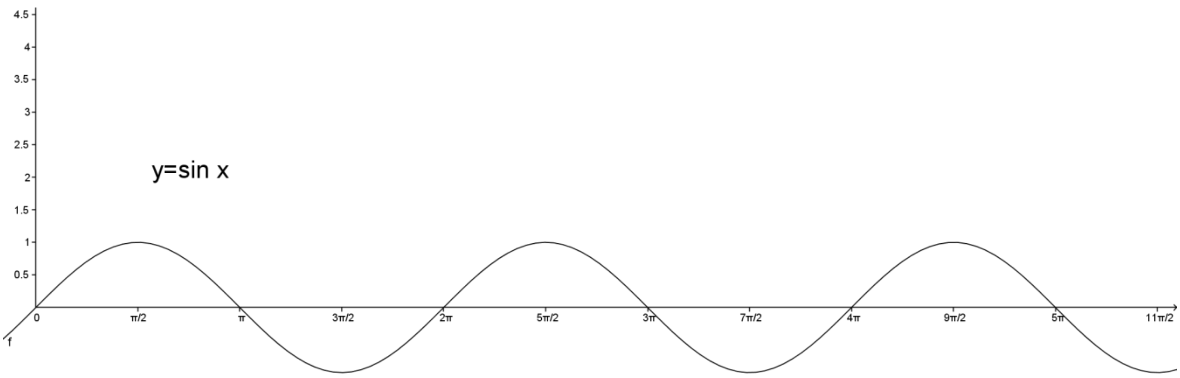
Trigonometry is a wide subject in mathematics and it is related to triangles, angles, sides and especially unit circle. When trigonometry topic is started to be discussed in courses, students usually generalize trigonometry as only ratios of angles. *"....Trigonometry is understood as relations between the angle and the edges of right triangle. For this reason, the students were successful on the questions concerned with trigonometry of angles. ... trigonometry is generally thought via teacher active method and memorizing the ready knowledge"* (Orhun, 2001) As the paper suggested by Nevin Orhun, teachers teach trigonometry with old fashioned methods like just giving the rules and solving examples of these rules. Then, when we come to the trigonometric functions and equations topic, students get in trouble to internalize the new concepts because they are stuck in angle concept. They couldn't go beyond higher level. The big reason behind the students' failure in trigonometric equations is that teachers do not know what students should know and where to start teaching topic. Therefore, the first important step is the awareness of teachers that how students learn trigonometric equations, what could cause the misunderstandings and misconceptions, and what information is needed to claim that the learning has occured.

Let us examine a trigonometric equation example to see where students have trouble to understand the concepts *(which means “zurna” says “zırt” ☺)*

There is an commonly used example “sin x= ½, x=?” at the beginning of trigonometric equations. As we mentioned above that relation between edges and right triangle comes first students’ mind, students would think the right triangle (30-60-90) and ratio of its edges. Since they have already learnt sin 30°= 1/2, they would say x=30. (not 30 degree because mostly students ignore units) As reported by Orhun (2001), when a trigonometry question is asked in terms of radian, compared to degree, number of students who make mistakes increases. We can deduce from this result that students have difficulty in radian concept. This fact is the reason of why most students say x=30° instead of $\pi/6$.

- Instead of asking the value of x directly, we want students to make connection between previous and former knowledge so it is better to build equation concept with functions. Hence, as a good start, we will ask students to draw sin x function. Since they have learnt it before, it will be good exercise to remind their previous knowledge too. (We will deal with positive x axis because we start with 0 degree in unit circle so we do not want to see negative angles)

Expected answer: It is important to see that students are able to draw sin x function correctly which means:

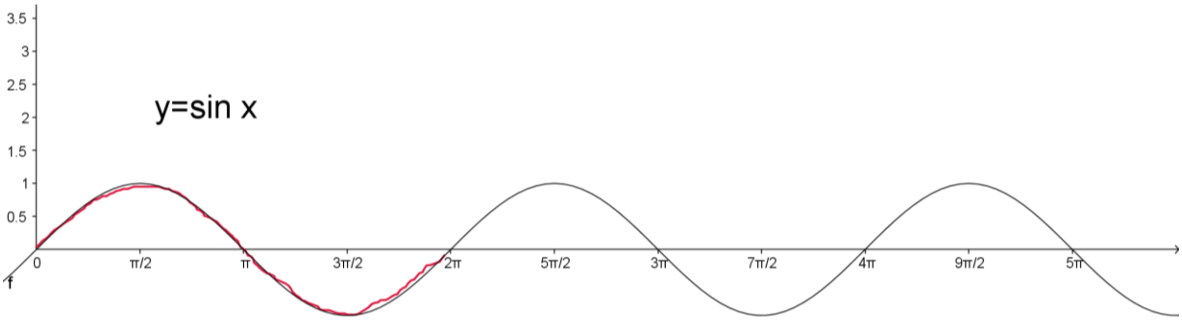


then some students may stop drawing at some point naturally. However, they should represent that sine x function continues through x axis like figure above. From this figure, we can make comment that students have intuition about infinity of sin x function.

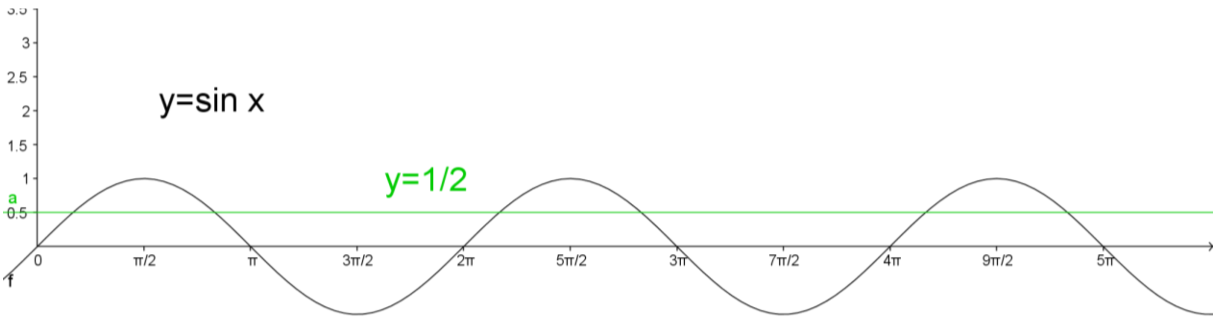
(NOTE: we have some contradiction about using the concept of inifinity here because students learn this concept in 12 th grade. Lets discuss it in the class.)

- It is useful to discuss some properties of sin x function after drawing.
  - ✓ Sin x is a periodic function (needed for solution set of trigonometric equation later)

    While discussing, students can claim that sin x is periodic. However, this can be only a memorization. Hence, students should be asked to show one period of sin x in the graph.

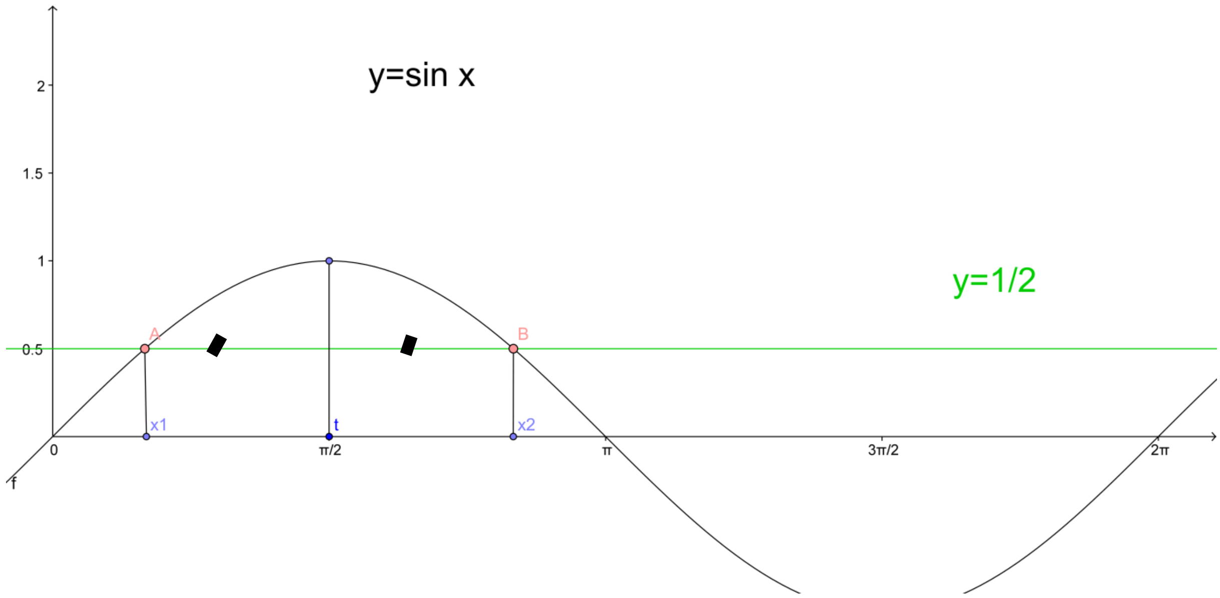


- In addition to sin x graph, we ask students to draw y = ½ line on the same graph. It can be asked that "Why did we draw two graphs together?" , because we want students to recognize intersection points. After students recognize the expected answer: intersection points, like sine function, students also should see from the graph that intersection points has no end because of the property of sine function. We will understand their awareness of ongoing intersection points if they show that in graph. Secondly, by periodic property of sine function, students should state that there are two intersection for each period.

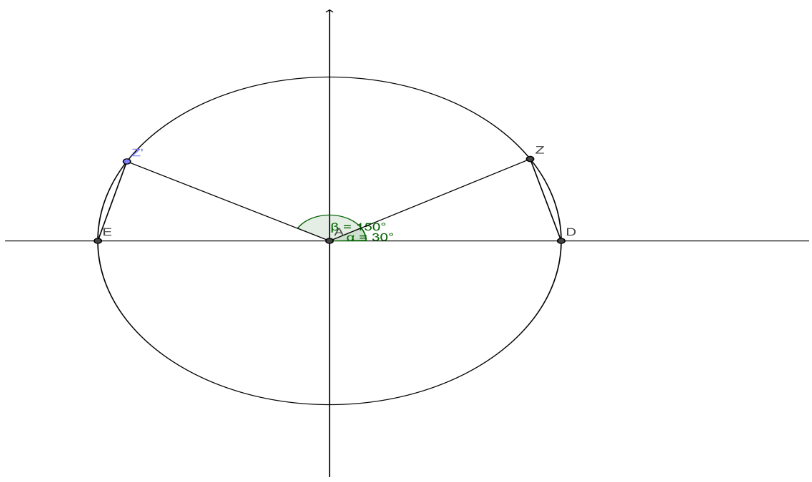


- Now it is time to determine the positive x values where the intersection occurs. We want students to mark intersection points on the graph. Then, we ask them to find x value of first intersection. If students say x = 30°, it is not enough to think that learning has occured. As a teacher, we should ask students why not 30° refers to second intersection. If they really are aware of what they are doing, they must answer that "because second intersection comes after the sin 90° =1, so 30° cannot come after 90°." Then, at that moment, they should be asked that "so determine the value of x at second intersection."

Discussion: If we consider the importance of connection between procedures, we decided to use parabola explanation to justify second x value is equal to 150°. According to parabola, a line passing through the peak point is accepted as symmetry line. Hence, distance between first x and intersection point, t, of symmetry line and x axis must be equal to distance between second x and t. Would you be convinced with this explanation, if you were 11th grade student?



Alternatively, if students explain why x =150° by using demonstration of sin x in unit circle, it is an acceptable answer but again we need one more question to understand whether students memorize knowledge or not. The question is "how did you get this solution?" One possible answer can be given by using similar triangles.



- After students find other angles of intersection points with the same process, we ask students to draw a table including angles we found and their sine values.

Sin 30°	1/2
Sin 150°	1/2
Sin 390°	1/2
Sin 510°	1/2
….	…

Then, we ask students what changes in this table? , answer would be of course "angles" Then, we can call that changing angles as "variable" and write our equation that we will ask for its solution set. We have all angles, now it remains to represent them in a more systematic way because we do not want that answer: {30°, 150°, 390°,510°…} We want students to come to this point:

$$S.S = \{x \mid x_1 = \alpha + 2k\pi;\ x_2 = (180 - \alpha) + 2k\pi\ ;\ k \in Z\}$$

Apart from 30° and 150°, we want students to show other angles on unit circle so that they will see relation between angles. For example, students will see that 30° +360°= 390° and 150°+360°= 510°. 30° is first intersection in first period and 150° is second intersection in first period. When we add 360° to 30°, we get first intersection in next period. So, adding 360° means turning a tour in unit circle. It is same for 150°. We manage students' thinking in this path. Finally, students will construct relation among x values such that there are x_1, x_2… where x_1= 30° , x_2= 150°, x_3= 30°+360°, x_4= 150°+360°, x_5= 30°+720° , …

x_1= 30° , x_2= 150°, x_3= 30°+360° ….. x_k =30°+ k.360°, x_{k+1}= 150°+k.360°, $k \in Z$

Now, it is time to set periodic relation between 30° and 150° with respect to180° (half of one period) => 30° = (180°-150°). Our final goal is to ask students to write these angles in terms of radian.

Note1: We did not specify an interval for solution set to examine general situation but in class, it can be given.

Note 2: we didn't choose sin x= 1 example because it doesn't matter 90° and (180°-90°) in solution set, as a result we cannot find general solution for other angles.

Note 3: We added the related pages of textbook at the end of our paper.

REFERENCES

Orhun, N., “Students’ mistakes and misconceptions on teaching of trigonometry”, The Mathematics Education into the 21st Century Project, International Conference on New Ideas in Mathematics Education, Palm Cove, Queensland, Australia, August 19-24, 2001.

Ministry of Education of Turkey, Highschool mathematics textbook, 2012

TRİGONOMETRİK DENKLEMLER

$\sin x = a$, $\cos x = a$, $\tan x = a$, $\cot x = a$ BİÇİMİNDEKİ TRİGONOMETRİK DENKLEMLER

Motivasyon

Dikdörtgen şeklindeki kitaplığın kenar uzunlukları $\tan x$ ve $\cot x$ birimdir. Kitaplığın çevre uzunluğu $\frac{8\sqrt{3}}{3}$ br ise x açısını nasıl bulunuz?

Etkinlik

(I) $\sin x = \frac{1}{2}$ (II) $\cos x = \frac{\sqrt{3}}{2}$ (III) $\tan x = \frac{1}{\sqrt{3}}$ (IV) $\cot x = \sqrt{3}$

- Yukarıdaki denklemleri inceleyiniz. Ölçüleri 30°, 150°, 210°, 330°, 390°, 510°, − 30°, −150°, −1470° olan açılardan hangilerinin bu denklemleri sağladığını bulunuz.
- (I) denklemini sağlayan açılar arasındaki ilişkiyi inceleyiniz. Bu açılardan başka sağlayan açılar var mıdır? Bu denklemin çözüm kümesinin genel olarak nasıl ifade edilebileceğini tartışınız.
- (I) denklemi için yaptığınız incelemeyi diğer denklemler için de yaparak çözüm kümelerini bulunuz.

➢ $\sin x = a$, $\cos x = a$, $\tan x = a$ ve $\cot x = a$ biçimindeki trigonometrik denklemlerin çözüm kümeleri için bir genellemeye ulaşmaya çalışınız.

Örnek

Aşağıdaki denklemlerin çözüm kümelerini bulalım.

a) $\cos x = \frac{1}{2}$ b) $\sin x = -\frac{1}{2}$ c) $\tan x = 1$ ç) $\cot x = -\frac{1}{\sqrt{3}}$

ÇÖZÜM

a) $\cos x = \frac{1}{2}$ denklemini sağlayan x değerlerinden biri 60° dir. x açısının kosinüsü 1 ve 4. bölgelerde aynı değerleri alacağından x açısının ölçüleri,

$x_1 = 60°, 420°, 780°, ...$

$x_2 = -60°, -420°, -780° ....$ dur. Buradan denklemin çözüm kümesi,

$Ç = \{x \mid x_1 = 60° + 360°.k \quad \vee \quad x_2 = -60° + 360°.k,\ k \in Z\}$ olarak yazılır.

b) $\sin x = -\frac{1}{2}$ denklemini sağlayan x değerlerinden biri 210° dir. x açısının sinüsü 3 ve 4. bölgelerde aynı değerleri alacağından x açısının ölçüleri,

$x_1 = 210°, 570°, 930°, \ldots$

$x_2 = -30°, -390°, -750°, \ldots$ olur. Buradan denklemin çözüm kümesi,

$Ç = \{x \mid x_1 = 210° + 360°.k \lor x_2 = -30° + 360°.k,\ k \in Z\}$ olarak yazılır.

c) $\tan x = 1$ denklemini sağlayan x değerlerinden biri 45° dir. x açısının tanjantı 1 ve 3. bölgelerde aynı değerleri alacağından x açısının ölçüleri,
$x = \ldots -135°, 45°, 225°, \ldots$ olur. Buradan denklemin çözüm kümesi,
$Ç = \{x \mid x = 45° + 180°.k,\ k \in Z\}$ olarak yazılır.

d) $\cot x = -\frac{1}{\sqrt{3}}$ denklemini sağlayan x değerlerinden biri 120° dir. x açısının kotanjantı 2 ve 4. bölgelerde aynı değerleri alacağından x açısının ölçüleri,
$x = \ldots -240°, 60°, 120°, 480°, \ldots$ olur. Buradan denklemin çözüm kümesi,
$Ç = \{x \mid x = 120° + 180°.k,\ k \in Z\}$ olarak yazılır.

Tanım ve Bilgi

1. $\sin x = \sin \alpha$ denkleminin çözüm kümesi,
$Ç = \{x \mid x_1 = \alpha + 360°.k \lor x_2 = 180° - \alpha + 360°.k,\ k \in Z\}$
$\sin(f(x)) = \sin(g(x))$ denklemin çözüm kümesi ise
$Ç = \{x \mid f(x) = g(x) + 360°.k \lor f(x) = 180° - g(x) + 360°.k,\ k \in Z\}$ dir.

2. $\cos x = \cos \alpha$ denkleminin çözüm kümesi,
$Ç = \{x \mid x_1 = \alpha + 360°.k \lor x_2 = -\alpha + 360°.k,\ k \in Z\}$
$\cos(f(x)) = \cos(g(x))$ denkleminin çözüm kümesi ise
$Ç = \{x \mid f(x) = g(x) + 360°.k \lor f(x) = -g(x) + 360°.k,\ k \in Z\}$ dir.

3. $\tan x = \tan \alpha$ ve $\cot x = \cot \alpha$ denklemlerinin çözüm kümesi,
$Ç = \{x \mid x = \alpha + 180°.k,\ k \in Z\}$
$\tan(f(x)) = \tan(g(x))$ ve $\cot(f(x)) = \cot(g(x))$ denklemlerinin çözüm kümesi ise,
$Ç = \{x \mid f(x) = g(x) + 180.k,\ k \in Z\}$ dir.

Örnek

1. $\cos(2x - 60°) = \sin x$ denkleminin gerçek sayılardaki çözüm kümesini bulalım.

ÇÖZÜM

$\cos(2x - 60°) = \sin x$ denklemini $\cos(2x - 60°) = \cos(90° - x)$ biçiminde yazabiliriz. Bu durumda,

$2x_1 - 60° = 90° - x_1 + 360°.k \quad \lor \quad 2x_2 - 60° = -(90° - x_2) + 360°.k$ yazılabilir.

$3x_1 = 150° + 360°.k \quad V \quad 2x_2 - 60° = -90° + x_2 + 360°.k$

$x_1 = 50° + 120°.k \quad V \quad x_2 = -30° + 360°.k$ bulunur.

Buradan denklemin çözüm kümesi,

$Ç = \{x \mid x_1 = 50° + 120°.k, \quad V \quad x_2 = -30° + 360°.k, \ k \in Z\}$ biçiminde yazılır.

2. $2\sin^2 x - 5\sin x + 2 = 0$ denkleminin $[0, 360°)$ aralığındaki çözüm kümesini bulalım.

ÇÖZÜM

$2\sin^2 x - 5\sin x + 2 = 0$ denkleminde $\sin x = t$ dönüşümü uygulayalım.

$2t^2 - 5t + 2 = 0$ denklemini çözersek,

$(2t - 1)(t - 2) = 0$ için $t_1 = \frac{1}{2}$ veya $t_2 = 2$ bulunur.

$\sin x = \frac{1}{2}$ ise; $\sin x = \sin 30°$ yazılır.

$x_1 = 30° + 360°.k \quad V \quad x_2 = 150° + 360°.k$ bulunur.

$\sin x = 2$ ise denklemin çözüm kümesi $\varnothing$ dir.

Sonuç olarak $2\sin^2 x - 5\sin x + 2 = 0$ denkleminin çözüm kümesi,

$Ç = \{x \mid x_1 = 30° + 360°.k \quad V \quad x_2 = 150° + 360°.k, \ k \in Z\}$ bulunur.

3. $\sin 4x + \sin 2x = \cos x$ denkleminin $[0,180°)$ aralığındaki çözüm kümesini bulunuz.

ÇÖZÜM

$\sin 4x + \sin 2x = \cos x$ denkleminde dönüşüm formülünü uygulayalım.

$$2\sin\frac{4x + 2x}{2}.\cos\frac{4x - 2x}{2} = \cos x$$

$2\sin 3x.\cos x = \cos x$

$2\sin 3x.\cos x - \cos x = 0$ denkleminde ortak paranteze alalım.

$\cos x(2\sin 3x - 1) = 0$

$\cos x = 0$ veya $\sin 3x = \frac{1}{2}$ denklemlerini çözelim.

a. $\cos x = 0$ ise $\cos x = \cos 90°$ denkleminde,

$x_1 = 90° + 360°.k \quad V \quad x_2 = -90° + 360°.k$ bulunur.

$k = 0$ için $x_1 = 90° \quad V \quad x_2 = -90° \notin [0,180°)$

$k = 1$ için $x_1 = 450° \notin [0,180°) \quad x_2 = 270° \notin [0,180°)$

$k \in Z$ için farklı değerlere karşılık $\cos x = 0$ denkleminin $[0,180°)$ aralığında tek kökü vardır. Bu durumda $Ç_1 = \{90°\}$ dir.

b. $\sin 3x = \frac{1}{2}$ ise $\sin 3x = \sin 30°$ denkleminde,

$3x_1 = 30° + 360°.k \quad V \quad 3x_2 = 180° - 30° + 360°.k$

$x_1 = 10° + 120°.k \quad V \quad x_2 = 50° + 120°.k$

$k = 0$ için $x = 10° \quad V \quad x = 50°$

$k = 1$ için $x = 130° \quad V \quad x = 170°$

$k = 2$ için $x = 250° \notin [0,180°)$

$k \in Z$ için bu denkleminde dört farklı kökü olur.

$Ç = \{10°, 50°, 130°, 170°\}$ bulunur.

Denklemin $[0, 180°)$ aralığındaki çözüm kümesi ise,

$Ç = Ç_1 \cup Ç_2 = \{10°, 50°, 90°, 130°, 170°\}$ şeklinde bulunur.

4. $3\sin^2x - \sin x\cos x - 4\cos^2x = 0$ denkleminin gerçek sayılardaki çözüm kümesini bulalım.

ÇÖZÜM

$3\sin^2x - \sin x\cos x - 4\cos^2x = 0$ denklemini çarpanlarına ayıralım.

$3\sin^2x - \sin x\cos x - 4\cos^2x = 0$

$3\sin x \qquad\qquad -4\cos x$

$\sin x \qquad\qquad +\cos x$

$(3\sin x - 4\cos x).(\sin x + \cos x) = 0$

$3\sin x - 4\cos x = 0 \quad \vee \quad \sin x + \cos x = 0$ denklemleri elde edilir.

a. $3\sin x - 4\cos x = 0$ denklemini çözelim.

$$\frac{3\sin x}{3\cos x} = \frac{4\cos x}{3\cos x}$$

$\tan x = \dfrac{4}{3} \Rightarrow x = \arctan\dfrac{4}{3}$ bulunur.

$Ç_1 = \{x \mid x = \arctan\dfrac{4}{3} + 180°.k,\ k \in Z\}$

b. $\sin x + \cos x = 0$ denklemini çözelim.

$$\frac{\sin x}{\cos x} = \frac{-\cos x}{\cos x}$$

$\tan x = -1$ ise $x = -45° + 180°.k$ bulunur.

$Ç_2 = \{x \mid x = -45° + 180°.k,\ k \in Z\}$

Buradan denklemin çözüm kümesi,

$Ç = Ç_1 \cup Ç_2 = \{x \mid x_1 = \arctan\dfrac{4}{3} + 180°.k \quad \vee \quad x_2 = -45° + 180°.k,\ k \in Z\}$ olur.

Uygulamalar

1. Aşağıdaki denklemlerin genel çözüm kümelerini yazınız.

a) $\tan x = \tan 20°$ b) $\sin x = \sin 40°$ c) $\cos x = \sin 50°$

ç) $\cot x = \tan 10°$ d) $\sin x = -\cos 2°$ e) $\cos x = \sin\left(-\dfrac{\pi}{3}\right)$

f) $\tan x = \tan\dfrac{\pi}{4}$ g) $\cos x = \sin\dfrac{\pi}{6}$ ğ) $\cot x = \tan(-40°)$

2. Aşağıdaki denklemlerin genel çözüm kümelerini yazınız.

a) $\cos 3x = \cos\left(x + \frac{\pi}{3}\right)$

b) $\sin 2x = \sin(x - 20°)$

c) $\tan 4x = \tan(-x)$

ç) $\cot(2x + 20°) = \cot x$

d) $\sin\left(3x - \frac{\pi}{5}\right) = -\sin x$

e) $\cos(4x - 60°) = \sin x$

f) $\tan\left(5x - \frac{\pi}{4}\right) = \cot 4x$

g) $\cot(3x - 10°) = \tan(-20°)$

3. Aşağıda verilen denklemlerin [0, 2π) aralığındaki çözüm kümelerini yazınız.

a) $\sin x = \sin \frac{3\pi}{5}$

b) $\cot x = \cot \frac{\pi}{3}$

c) $\cos x = \cos \frac{\pi}{8}$

ç) $\sin x = \cos 20°$

d) $\cot x = \tan 40°$

e) $\sin x = \cos 20°$

f) $\sin 2x = \sin(x + 135°)$

g) $\sin 2x = \sin 5x - \sin 3x$

ğ) $\tan \frac{3x}{4} = \cot \frac{\pi}{6}$

h) $\cos(3x - 40°) = \sin(x + 45°)$

ı) $\cos 2x - 4\sin x - 3 = 0$

i) $\cos 5x.\cos 3x = \cos 6x.\cos 2x$

4. Aşağıda verilen denklemlerin çözüm kümelerini bulunuz.

a) $\tan(\pi + 2x) = \cot 3x$

b) $\sin 7x = \cos\left(x + \frac{\pi}{4}\right)$

c) $\sin 4x + \sin 2x = 0$

ç) $\cos 5x.\cos 3x - \sin 5x.\sin 3x = \frac{\sqrt{3}}{2}$

d) $\sin 8x.\sin 2x = \sin 7x.\sin 3x$

e) $\arctan x + \arctan(1 - x) = \arctan \frac{4}{3}$

f) $\tan 3x.\cot x = 1$

5. $\sin\left(\frac{\pi}{6} + x\right) - \cos\left(\frac{\pi}{3} + 2x\right) = 0$ denklemini sağlayan en küçük pozitif x açısı kaç derecedir?

6. $\cot x - 2\cos^2 x = 0$ denkleminin $[0, 2\pi)$ aralığındaki çözüm kümesini bulunuz.

$a \cos x + b \sin x = c$ BİÇİMİNDEKİ TRİGONOMETRİK DENKLEMLER

Etkinlik

$\sin x - \sqrt{3}.\cos x = 1$ denklemini çözmeye çalışınız.

- $\sqrt{3}$ yerine $\tan 60^\circ = \dfrac{\sin 60^\circ}{\cos 60^\circ}$ yazınız.
- Denklemi düzenleyerek toplam ve fark formüllerini kullanınız.
- Önceki kazanımlardan öğrendiğiniz bilgilerden yararlanarak denklemi çözmeye çalışınız.

☞ $a.\cos x + b.\sin x = c$ biçiminde verilen denklemleri çözümünde nasıl bir yol izleyebileceğinizi tartışınız.

Örnek

$\cos x + \sin x = 1$ denkleminin gerçek sayılardaki çözüm kümesini bulalım.

ÇÖZÜM

$\cos x + 1.\sin x = 1$ denkleminde $\tan 45^\circ = 1$ değerini yerine yazalım.

$\cos x + \tan 45^\circ.\sin x = 1$

$\cos x + \dfrac{\sin 45^\circ}{\cos 45^\circ}.\sin x = 1$ eşitliğinde payda eşitlersek,

$\dfrac{\cos x.\cos 45^\circ + \sin 45^\circ.\sin x}{\cos 45^\circ} = 1$ olur. Fark formülünden,

$\cos(x - 45^\circ) = \cos 45^\circ$ denklemi elde edilir. Denklemde,

$x - 45^\circ = 45^\circ + 360^\circ.k$ veya $x - 45^\circ = -45^\circ + 360^\circ.k$

$x_1 = 90^\circ + 360^\circ.k$ veya $x_2 = 0 + 360^\circ.k$ bulunur.

$Ç = \{x \mid x_1 = 90^\circ + 360^\circ.k \quad \vee \quad x_2 = 360^\circ.k,\ k \in Z\}$ olur.

Tanım ve Bilgi

$a, b, c \in R$ olmak üzere,

$a.\cos x + b.\sin x = c$ biçimindeki denklemlere doğrusal denklem denir. Bu tür denklemler $\cos x + \dfrac{b}{a}.\sin x = \dfrac{c}{a}$ şeklinde düzenlenip $\dfrac{b}{a} = \tan \alpha$ değeri yerine yazıldıktan sonra çözüm kümesi bulunur.

Örnek

$3\cos x - \sqrt{3}\sin x = 2\sqrt{3}$ denkleminin gerçek sayılardaki çözüm kümesini bulalım.

ÇÖZÜM

$3\cos x - \sqrt{3}\sin x = 2\sqrt{3}$ denkleminde eşitliğin her iki tarafını üçe bölelim. Denklem bu durumda,

$\cos x - \frac{\sqrt{3}}{3}\sin x = \frac{2\sqrt{3}}{3}$ şeklinde olur.

$\frac{\sqrt{3}}{3} = \tan 30°$ değerini yerine yazalım.

$\cos x - \tan 30°.\sin x = \frac{2\sqrt{3}}{3}$

$\cos x - \frac{\sin 30°}{\cos 30°}.\sin x = \frac{2\sqrt{3}}{3}$ eşitliğinde payda eşitleyelim.

$\cos x.\cos 30° - \sin 30°.\sin x = \frac{2\sqrt{3}}{3}.\cos 30°$

$\cos(x + 30°) = \frac{2\sqrt{3}}{3}.\frac{\sqrt{3}}{2}$

$\cos(x + 30°) = 1$ bulunur.

$\cos(x + 30°) = \cos 0°$ şeklinde yazarak denklemi çözebiliriz.

$x_1 + 30° = 0° + 360°.k \quad \vee \quad x_2 + 30° = -0° + 360°.k$

$x_1 = x_2 = -30° + 360°.k$ bulunur.

$Ç = \{x \mid x = -30° + 360°.k, k \in Z\}$ olur.

Uygulamalar

1. $\sin x + \sqrt{3}.\cos x = 4$ denkleminin çözüm kümesini bulunuz.
2. $a.\sin x + \cos 2x = 1$ denkleminin çözüm kümesinin bir elemanı $60°$ ise a kaçtır?
3. $\frac{3}{\cos x} = \frac{4}{\sin x}$ olduğuna göre cosx in pozitif değerini bulunuz.
4. $\cos x + \sqrt{3}.\sin x - \sqrt{3} = 0$ denkleminin $[0, \pi]$ aralığındaki köklerini bulunuz.
5. $x.\cos 40° - \sin 40° = x$ ifadesinde x in $(-\cot 20°)$ ye eşit olduğunu gösteriniz.
6. $a = -5.\sin x + 12.\cos x$ ise a nın alabileceği en küçük değerini bulunuz.
7. $2.\sin\alpha + \cos^2\alpha - \sin^2\alpha$. $\sin 2\alpha = \frac{\sqrt{2}}{3}$ ise $\tan 2\alpha$ neye eşittir?
8. $2.\sin^2 x + 3.\sin 2x = 4$ eşitliği veriliyor. Buna göre tan x in alacağı değerler toplamını bulunuz.